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To verify this, the incident power P_i supplied to the cavity was varied by means of a calibrated microwave attenuator and the corresponding critical dc potential across the cavity V_c below which the avalanche started was noted in each case. The experimental results (shown in figure 2) obtained at pressures of 5×10^{-4} torr show the dependence

$$P_i \propto V_c^2.$$

As another test the microwave power necessary to initiate the electron avalanche was measured as a function of pressure between 5×10^{-5} torr and 5×10^{-3} torr. The results are shown in figure 3.

A detailed theoretical description of the avalanche process (to be published) correctly predicts the square law dependence and the minimum in the power against pressure curve as obtained in the experiment. Further experimental and theoretical work on the avalanche effect is in progress.

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Instabilities in perpendicular collisionless shock waves

Abstract. A comparison of the ion acoustic instability with the $\boldsymbol{E} \times \boldsymbol{B}$ electron drift instability leads to predictions concerning observations of enhanced fluctuations in perpendicular collisionless shock waves.

There have been several observations of enhanced fluctuations within shock waves propagating perpendicular to an external magnetic field in a plasma (Paul *et al.* 1969, Daughney *et al.* 1970, Chodura *et al.* 1970). Several theoretical investigations stimulated by the papers of Krall and Book (1969 a, 1969 b) have examined the role of the $\boldsymbol{E} \times \boldsymbol{B}$ electron drift instability as a source of these enhanced fluctuations. The basic picture consists of magnetized electrons drifting relative to unmagnetized ions; only electrostatic waves are considered. The electric field arises owing to charge separation in the shock.

Wong (1970) was the first to obtain a correct solution of the linear dispersion relation for this instability, in the limiting case of zero ion temperature and propagation perpendicular to the magnetic field. Biskamp (1970) was the first to point out that

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this instability propagated only in a narrow cone within a few degrees of the direction perpendicular to $\boldsymbol{B}_0$. Independently, Gary and Sanderson (1970) and Gary (1970) did a thorough analysis of this instability, considering off-perpendicular propagation and thermal ions with $T_e \gg T_i$. More recently, Lampe and Orens (1970) have done an exhaustive study of the perpendicular, cold ion case.

When $T_e \gg T_i$, ion Landau damping is weak so that an electron drift velocity which is greater than the phase velocity of the ion acoustic waves but less than the electron thermal velocity v_e is sufficient to excite unstable waves. The instability is of the two-stream type and appears at 'crossings' in the ω–k plane of the lightly damped ion acoustic wave with the Doppler-shifted electron Bernstein modes (Gary and Sanderson 1970).

When $T_e \sim T_i$, however, ion Landau damping is comparatively strong, and electron drift velocities greater than v_e are necessary to make ion acoustic waves unstable (Stringer 1964). For drift velocities less than v_e, the 'crossing' is no longer a source of instability.

Lashmore–Davies (1970) pointed out, however, that there is another mechanism associated with the $\boldsymbol{E}\times\boldsymbol{B}$ electron drift which allows instability when $T_e \sim T_i$ and $v_0 \ll v_e$. This is due to the fact that the Doppler-shifted electron Bernstein mode 'sees' a positive slope to the ion distribution function and thereby undergoes inverse Landau damping. Recently, Forslund *et al.* (1970) observed this resonant instability in a computer simulation and carried through a numerical analysis of the linear dispersion relation in the case of perpendicular propagation. Their results show clearly that the maximum growth rate occurs near the crossing of the electron Bernstein mode with the line $\omega/k = \sqrt{2}v_i$, rather than at the ion acoustic phase velocity as in the $T_e \gg T_i$ case.

The purpose for this letter is to derive an expression for the maximum growth rate for this instability, to compare it with the ion acoustic instability, and to examine the experimental implications of this comparison.

A dispersion relation for electrostatic waves in a Vlasov plasma of unmagnetized ions and magnetized electrons is derived in Gary and Sanderson (1970). In the particular case of electrons undergoing an $\boldsymbol{E}\times\boldsymbol{B}$ drift $\boldsymbol{v}_0$, and wave propagation perpendicular to the applied magnetic field $\boldsymbol{B}_0$, there is

$$1+\bar{K}_i+\bar{K}_e = 0 \tag{1}$$

where

$$\bar{K}_i = -\frac{k_i^2}{2k^2} Z'\left(\frac{\omega}{\sqrt{2}kv_i}\right)$$

and

$$\bar{K}_e = \frac{k_e^2}{k^2}\left\{1-\exp\left(-\frac{k^2v_e^2}{\Omega_e^2}\right)I_0\left(\frac{k^2v_e^2}{\Omega_e^2}\right) -2(\omega-kv_0)^2 \sum_{m=1}^{\infty} \frac{\exp(-k^2v_e^2/\Omega_e^2)I_m(k^2v_e^2/\Omega_e^2)}{(\omega-kv_0)^2-m^2\Omega_e^2}\right\}.$$

Here $v_j^2 = \omega_j^2/k_j^2 = T_j/m_j$ for species j and $\Omega_e = eB_0/m_ec$. Assuming that a particular term in the sum is dominant, and that $kv_e/\Omega_e \gg 1$, the dispersion relation is

$$1-\frac{k_e^2}{2k^2}\frac{T_e}{T_i}Z'\left(\frac{\omega}{\sqrt{2}kv_i}\right)+\frac{k_e^2}{k^2}\left(1-\frac{2(\omega-kv_0)^2}{(\omega-kv_0)^2-m^2\Omega_e^2}\frac{\Omega_e}{(2\pi)^{1/2}kv_e}\right) = 0. \tag{2}$$

Assume $\omega = \omega_R + \delta\omega + i\gamma$ and that $\omega_R - kv_0 = \pm m\Omega_e$. Then, assuming both $|\delta\omega|$ and $|\gamma|$ are small compared with $|\omega_R - kv_0|$,

$$\epsilon \equiv 1 + \frac{k^2}{k_e^2} - \frac{T_e}{2T_i} Z'\left(\frac{\omega}{\sqrt{2}kv_i}\right) \cong \frac{(\omega_R - kv_0)}{\delta\omega + i\gamma} \frac{\Omega_e}{(2\pi)^{1/2}kv_e} \tag{3}$$

and the maximum growth rate is

$$\gamma_{max} \cong \frac{(kv_0 - \omega_R)}{|\epsilon|^2} \frac{\Omega_e}{(2\pi)^{1/2}kv_e} \operatorname{Im}\left\{-\frac{T_e}{2T_i} Z'\left(\frac{\omega}{\sqrt{2}kv_i}\right)\right\}. \tag{4}$$

Now using the result (Lashmore–Davies 1970, Forslund *et al.* 1970)

$$\frac{\omega_R}{k} = \sqrt{2}v_i \tag{5}$$

there is

$$\frac{\gamma_{max}}{\Omega_e} \simeq (0{\cdot}368)\frac{T_e}{T_i}\left(\frac{v_0 - \sqrt{2}v_i}{\sqrt{2}v_e}\right) \Big/ \left\{\left(1 + \frac{k^2}{k_e^2} - 0{\cdot}075\frac{T_e}{T_i}\right)^2 + 0{\cdot}425\left(\frac{T_e}{T_i}\right)^2\right\}. \tag{6}$$

This expression provides substantially better agreement with the results of Forslund *et al.* than does Lashmore–Davies' equation.

The most important differences between the $\boldsymbol{B}_0 = 0$ ion acoustic instability (IAI) and the $\boldsymbol{E}\times\boldsymbol{B}$ electron drift instability (EBI) may be summarized:

(i) For a given v_0/v_e, EBI generally has the larger growth rate, and can be unstable for T_e/T_i values such that IAI is stabilized.

(ii) The cone of propagation of EBI is generally much narrower than that of IAI. (See for example figure 7 of Gary 1970). If $\hat{k}\,.\,\hat{v}_0 = \cos\Theta$, where Θ lies in the plane of $\boldsymbol{v}_0$ and $\boldsymbol{B}_0$, EBI must satisfy (see equation (16) of Gary (1970)):

$$\sin\Theta < \frac{\Omega_e}{\sqrt{2}kv_e} \tag{7}$$

whereas the comparable IAI condition in the $T_e \gg T_i$ limit is

$$\cos\Theta > \frac{(T_e/m_i)^{1/2}}{v_0} \tag{8}$$

(iii) The phase velocity of EBI for $T_e \sim T_i$, equation (5) above, is smaller than the ω/k for both IAI and EBI at $T_e \gg T_i$ (Gary and Sanderson 1970 equation (8)):

$$\frac{\omega_R}{k} = \left(\frac{T_e}{m_i}\right)^{1/2}\left(\frac{3T_i}{T_e} + \frac{1}{1 + k^2/k_e^2}\right)^{1/2}. \tag{9}$$

To discuss the experimental implications of these results we compare two experiments, the 'Tarantula' linear pinch at Culham Laboratory, Great Britain (Paul *et al.* 1969, Daughney *et al.* 1970) and a theta pinch experiment at the Institut für Plasmaphysik, Garching, Germany (Keilhacker *et al.* 1969, Kornherr 1970, Chodura *et al.* 1970).

In the Culham experiment the shock propagates into a $T_e = T_i$ plasma; its passage heats the electrons so that toward the rear of the shock (which is presumably where the enhanced fluctuations are observed) $T_e \gg T_i$. Present Culham results are insufficient to allow a conclusion as to which instability is more important in the development of the shock. Laser scattering from the fluctuations indicate their phase velocity is consistent with equation (9), but at $T_e \gg T_i$ this cannot distinguish between IAI and EBI. The linear growth rates of EBI can easily account for the observed level of fluctuations, but so can the IAI if one calculates v_0 from the observed potential rise in the shock front. Also, scattering in the plane perpendicular to $\boldsymbol{B}_0$ has shown enhanced fluctuations out to about 55° from $\boldsymbol{v}_0$ (Paul *et al.* 1970); this is consistent with equation (8) which is satisfied by both instabilities in the plane perpendicular to $\boldsymbol{B}_0$.

The present most likely picture of the Culham shock is this: EBI turns on early in the shock, heating the electrons so that $T_e \gg T_i$. The ion acoustic waves then go unstable, and, because they are unstable over a much wider cone about the drift velocity, dominate the enhanced fluctuations at the rear of the shock. Thus, if a scattering experiment could be done to observe fluctuations in the $\boldsymbol{v}_0$–$\boldsymbol{B}_0$ plane, we should expect enhanced fluctuations to be observed through a broad cone—of the order of the 55° observed in the perpendicular direction.

In the Garching experiment the initial plasma satisfies $T_i > T_e$ and, although the shock heats electrons, the condition $T_e \sim T_i$ obtains at the rear of the shock, where enhanced fluctuations are observed.

In this case it is clear that IAI cannot be responsible for the enhanced fluctuations, and that EBI plays an important role. Thus we expect waves in the $\boldsymbol{v}$–$\boldsymbol{B}_0$ plane to follow equation (7). So, if nonlinear effects are unimportant, enhanced fluctuations for $kv_e/\Omega_e \gg 1$ should appear only at small angles (say less than 10°) to the drift velocity.

In summary: In the Culham perpendicular shock, $T_e \gg T_i$, ion Landau damping is weak, and we expect to see strong enhanced fluctuations away from the perpendicular to $\boldsymbol{B}_0$. In the Garching experiment, $T_e \sim T_i$, ion Landau damping is strong, and we expect the enhanced fluctuations to be concentrated within a few degrees of the perpendicular to $\boldsymbol{B}_0$.

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The electric charge of interacting cosmic ray particles at sea level

Abstract. The magnitude of the electric charge of sea level cosmic ray particles producing energy transfers greater than or approximately equal to 25 GeV in a 22·9 cm thick steel target have been measured. The charge distribution shows a peak at the electron charge and an apparent high charge tail. Spurious effects contributing to the high charge tail are knock-on electrons, and nuclear interactions in the proportional counter walls, which are used to measure the charge. The limit obtained on the flux of $4e/3$, $5e/3$ and $7e/3$ quarks is less than $8{\cdot}2\times10^{-10}\ \mathrm{cm^{-2}\ s^{-1}\ sr^{-1}}$.

The possibility of stable compound states of quarks of charge $4e/3$, $5e/3$ etc. was first proposed by de Swart (1967) although there have been few experiments to search for such objects in cosmic rays. The present experiment was undertaken to search for $4e/3$, $5e/3$ and $7e/3$ quark states in cosmic rays and is part of a program at Durham in which a systematic search is being made for quarks in cosmic rays.

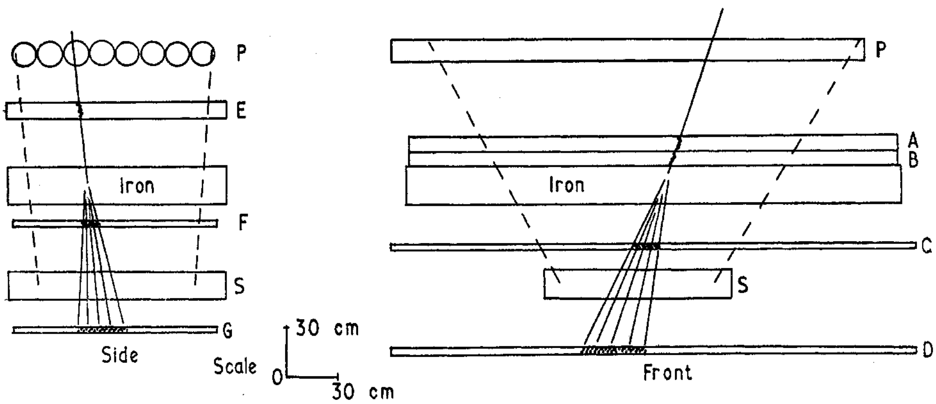

Figure 1. Experimental arrangement. P—proportional counters; A,B,C,D,E, F,G—flash tubes; S—scintillation counter.

The apparatus used is shown in figure 1 and the idea was to select events in which an incident charged particle traversed the proportional counter layer and produced an interaction in the steel target of energy transfer greater than 25 GeV. The known particles producing such events are high energy protons, pions and muons and such triggers enabled the most probable pulse height produced by high energy particles of charge e to be continuously monitored throughout the experiment.